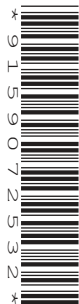


Cambridge International AS & A Level

THINKING SKILLS**9694/31**

Paper 3 Problem Analysis and Solution

May/June 2026**2 hours**

You must answer on the enclosed answer booklet.

You will need: Answer booklet (enclosed)
Calculator

INSTRUCTIONS

- Answer **all** questions.
- Follow the instructions on the front cover of the answer booklet. If you need additional answer paper, ask the invigilator for a continuation booklet.
- You should use a calculator where appropriate.
- Show your working.
 - Where a final answer is incorrect or missing, you may still be awarded marks for correct steps towards a solution.
 - In some questions, if you do not show your working, full marks will not be awarded.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages. Any blank pages are indicated.

- 1 The pirate Whitebeard needs to provide the secret location of buried treasure to his son, Blackbeard. He uses a grid to substitute a letter for each of the six digits of the map reference of the location, using one column per digit. His assistant, Stubble, produced the grid below for him to use.

	1st	2nd	3rd	4th	5th	6th
0	Q	F	O	C	X	E
1	T	D	I	X	M	Z
2	B	W	F	T	Z	G
3	G	E	Y	V	L	A
4	E	V	A	K	Z	W
5	J	R	V	L	F	C
6	N	B	Y	M	X	Q
7	U	H	D	P	F	I
8	A	P	S	O	H	G
9	K	W	U	J	R	N

For example, the location 014674, which is 14 metres east and 674 metres north of a fixed location, would be encoded using this grid as **QDAMFW**.

- (a) What is the code for 434297 using this grid? [1]

Blackbeard received the encoded location **GVYXXG**, but did not find any treasure where he looked. He was also annoyed to realise that Stubble's grid was faulty, because the code could refer to more than one map reference.

- (b) How many map references might **GVYXXG** refer to? Identify **one** of them. [2]

- (c) What is the maximum number of map references that could all have the same code using Stubble's grid? [2]

Whitebeard told Stubble to produce a new grid that was not faulty. Whitebeard also told Stubble to arrange the letters so that a repeated digit in a map reference never resulted in a repeated letter in the code.

Whitebeard used this new grid to encode the location of the buried gold as **SJDRGF**, the silver as **ZYDRGF**, and the bronze as **ZYDLGL**.

- (d) These three locations lie on three of the corners of a rectangle. What is the code for the location of the fourth corner of this rectangle? [1]
- (e) (i) What is the minimum possible distance between the gold and the silver? [2]
- (ii) What is the maximum possible distance between the silver and the bronze? [2]

Turn over

- 2 Three contestants take part in each episode of the TV general knowledge programme 'Kwizztime'. Each of them builds up their own 'cashpot', but only one of them gets the chance to take home its contents.

In the first round, the contestants are asked 20 questions each. Every correct answer puts \$400 into their own cashpot, but every incorrect answer puts \$200 into the cashpots of each of their opponents. Responses such as 'don't know' or 'pass' count as incorrect answers. Additionally, the fastest to answer their 20 questions receives a bonus of \$750 and the slowest has \$500 subtracted.

- (a) What is the largest total that a contestant could possibly have in their cashpot at the end of the first round of the show? [2]

At the end of the first round, the contestant with the smallest cashpot leaves the show.

In the first round of today's show, Sheila answered her 20 questions first, followed by Lionel, and finally Alison. The results were as follows:

contestant	time taken	correct answers	cashpot
Sheila	160 seconds	17	\$9600
Lionel	171 seconds	15	\$7900
Alison	154 seconds	11	\$6750

- (b) (i) What is the minimum total number of correct answers that Alison would have needed for her to progress to the second round? [2]
- (ii) If Alison had taken more time to give her answers, she might have got more correct answers.

What is the minimum total number of correct answers she would have needed to guarantee that she progressed to the second round? [3]

In the second round, the two remaining contestants are asked 20 further questions each. In this round each correct answer puts \$400 into the contestant's cashpot, but there are no penalties for incorrect answers. However, \$5 is subtracted from the contestant's cashpot for each complete second of the total time taken to answer their 20 questions.

Sheila has already answered her 20 questions in today's second round and she has increased her cashpot to \$15 200. She took exactly the same total time to answer them as in the first round.

- (c) (i) How many correct answers did Sheila give in today's second round? [2]
- (ii) Even if Lionel could answer all of his 20 questions correctly, what is the longest total time he could take to answer them in order to exceed Sheila's cashpot total? [2]

The contestant with the greater amount in their cashpot at the end of the second round progresses to the final part of the show, known as 'Cashpot Collection'. The contestant is asked an unlimited number of questions, and wins the current value of their cashpot at the moment they give their fifth correct answer. However, the value of the cashpot decreases by \$500 every time an incorrect answer is given and by \$100 every second after 50 seconds.

In yesterday's Cashpot Collection, Elliot won nothing despite having \$14 800 in his cashpot at the end of the second round. When his cashpot reached \$0, he had answered only 2 questions correctly in a total of 133 seconds.

(d) How many questions had Elliot been asked during his Cashpot Collection attempt? [2]

In a recent show, Celia won the highest amount ever on Kwizztime. At the end of the first round her cashpot was \$12 750. In the second round she gave 16 correct answers in 144 seconds. In the Cashpot Collection she took a total of 63 seconds, during which she gave 3 incorrect answers.

(e) How much did Celia win? [2]

- 3 The village council at Urmitidge meets to discuss issues that affect the small community. There is one field on the edge of the village that is owned communally, and rarely used, and some of the villagers think it should be sold to raise money for the community. There are two options being considered:

- Selling the field to a construction company to build houses
- Selling the field to a leisure company to build a golf course

A survey of the 22 villagers reveals that:

- 8 of the villagers support both options, but prefer a golf course
- 1 of the villagers supports both options, but prefers houses
- 4 support the golf course but not houses
- 5 support houses but not the golf course
- 2 are against selling the field
- the remaining 2 do not have an opinion

The council wants to hold ballots in which the villagers will vote to decide which, if either, of the two options should be chosen. The questions in the ballots will each have two possible answers. To win a ballot, an answer must receive **at least two-thirds** of the votes cast in response to that question. If neither answer in a ballot wins then that ballot is said to be 'unresolved'.

(a) Initially, the council considers holding two separate ballots:

- 1 'Do you support selling the field to build houses?'
- 2 'Do you support selling the field to build a golf course?'

(i) Determine whether each of these ballots would be won by 'Yes', if only the 20 villagers who have an opinion voted. [2]

(ii) Some villagers wonder whether, if these ballots were held, it would be worth trying to persuade one or both of the villagers who do not have an opinion to vote.

Explain how such additional votes could change the results of these ballots. [2]

(b) Another idea is to hold a pair of linked ballots. The second ballot would be held only if the first was won by 'Yes', and would be voted on only by those who had voted 'Yes'.

- 1 'Do you support selling the field?'
- 2 'Do you prefer to sell it to build houses or a golf course?'

If only the 20 villagers who have an opinion voted, which, if either, of the options would be chosen? Explain your answer. [2]

- (c) Finally, the council decides to hold a different pair of linked ballots. The second ballot would be held only if the first was won by either of the answers.

- 1 'If the field were to be sold, would you prefer to build houses or a golf course?'
- 2 'Do you support the option that was most popular in the first ballot?'

The two villagers who do not want the field to be sold are thinking about how they could prevent it from happening. They assume that the two villagers who do not have an opinion would not vote in these ballots.

- (i) Explain how the two villagers who do **not** want the field to be sold can prevent it from happening, by voting in only the second ballot. [2]

In fact, one of the villagers who does not have an opinion has been persuaded to support the golf course.

- (ii) How must the two villagers who do **not** want the field to be sold now vote to prevent it from happening? Explain your answer. [2]

- 4 Jason owns a small company which produces and sells wooden toys in three different sizes: small, medium and large. There are three stages in the production of each toy: cutting out, assembly and decoration, in that order. The times taken to complete each stage for each size of toy are given in the table.

size of toy	time (minutes)		
	cut out	assemble	decorate
small	5	35	20
medium	10	40	25
large	15	45	40

Alfie, Brianne and Colin are employees in Jason's company. Each employee works 8 hours a day, from 09:00 to 18:00 with a lunch break from 13:00 to 14:00, five days a week from Monday to Friday. Alfie produces the small toys, Brianne produces the medium toys and Colin produces the large toys. Each day the production is continuous during working hours, and a toy started before lunch can be continued after lunch. A toy may be incomplete at the end of a day, except on Fridays when a toy will not be started if it cannot be finished by the end of Friday.

- (a) How many of each size of toy are produced in a week? [2]

A customer wants an equal number of toys of each size, to be produced next week. Jason decides to produce these toys himself. He will work the same hours and with the same restriction about the completion of toys as his employees.

- (b) What is the maximum number of toys that Jason can produce on his own for this customer next week? [2]

A second customer wants only small and medium toys, to be produced next week. He wants twice as many small toys as medium toys. Alfie will produce these toys instead of what he usually does.

- (c) What is the maximum number of toys of each size that Alfie can produce on his own for this second customer next week? [1]

Jason would like to improve the quality of the toys. He has noted the particular skills of his employees, and he decides to change the way in which they work. Now, for all three sizes of toy, Alfie will cut out, Brianne will assemble and Colin will decorate. No employee will start their stage of the production unless they have time to finish it on the same day.

Each employee will complete their stage of the production in the order small, medium, large, small, medium, large and so on. There must be no partially complete toys at the beginning of any week.

- (d) (i) Show that Alfie can cut out 48 toys on a Monday. [1]
- (ii) Jason thinks that Brianne will be able to assemble 4 toys of each size on a Monday. Explain why he is **not** correct. [1]
- (iii) Show that the third toy produced on a Monday will be completed at 11:45. [3]

Jason realises that he could make the production of toys more efficient by having only two employees involved in making each toy. Alfie will cut out the toy, Brianne will assemble it and then Alfie will decorate it. Alfie will decorate toys as soon as they are assembled, but will continue to cut out toys if there are none ready to be decorated.

Jason tests this out on a working day of 8 hours with **small** toy production.

- (e) (i) How many small toys will be completed by the end of the day? [3]
- (ii) What is the largest number of small toys that Alfie could have cut out that will **not** have been assembled or decorated by the end of the day? [2]

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